EXPLORING LEARNING TRAJECTORIES IN ALGEBRA: INFORMING THE TEACHING OF FUNCTIONS

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Extended abstract

When teaching functions, mathematics teachers often know the outcomes that their learners are expected to achieve, but do not necessarily know the best way in which to teach to ensure that their learners achieve these outcomes. Learners are not able to grasp a concept in its entirety after seeing it only once. The pathway to understanding is a long one, and it takes time to learn a concept.

Learning trajectories chart the paths of learning, instead of just looking at the outcome of learning. The South African curriculum states its learning objectives in terms of outcomes, but does not say how these outcomes should be reached. It seems that many school curriculums are similar, and hence the path of learning throughout a topic or concept is not chartered. There is no mention of strategies, or thinking that learners use in order to achieve the required outcomes in the curriculum. This leaves teachers free to figure out the best way to achieve these outcomes (Daro, Mosher & Corcoran, 2011). The trade-off for autonomy in reaching these outcomes may be uncertainty on the part of the teachers, as they may not be able to, or do not have the time to devise the best paths to these outcomes.

Learning trajectories, especially in mathematics, is an area of research which focuses on learners' progression in their thinking about concepts. Wilson, Mojica and Confrey (2013) suggest that learning trajectories are very useful in helping teachers to understand how learners progress in their mathematical thinking, a process which may improve teaching practices.

Ronda (2004) has created an empirically based conceptual framework which describes learners' development of their understanding of functions, and refers to this path of development as learning trajectories. This *Framework of Growth Points* maps out 'big ideas' which learners typically encounter on their path to understanding functions (Ronda, 2009: p. 31). These growth points are in approximately the order that they are expected to encounter these ideas. Ronda (2009) suggests that most learners follow learning paths or trajectories, and reach growth points, as they develop their understanding of 'big ideas' of functions. My research uses Ronda's (2004) study into growth points and their description as its basis.

In testing Grade 9, 10 and 11 learners from a typical school in South Africa, I aimed to find out about learning trajectories in functions, and particularly in the area of functions when they are represented by an equation. I have used Ronda's (2004) framework of growth points to guide this research, and compare the way in which learners in South Africa progress through the growth points set out by Rhonda in her study which was done in Australia and the Philippines.

My research also deals with the language associated with the growth points, and how learners' language and use of language changes as they progress along their path of understanding. Caspi and Sfard's (2012) describe the progression through levels of discourse as canonic. This means that there is a hierarchy of levels of algebraic discourse, where each level of discourse builds on the previous level, and is hence more complex than the previous level (a meta-discourse of the previous level). Caspi and Sfard (2012) state that "transition from one level to another can be seen as developmental milestones", and hence this links with meeting Ronda's growth points on a learning trajectory.

In this paper, I will show that the learning of functions is not a quick process where an outcome can be achieved, but rather a process whereby learners reach growth points along which they progress systematically.

I also hope to show that certain types of language can be associated with the growth points. Teachers will be able to use this to better inform their teaching of functions to learners.

REFERENCES

Caspi, S. & Sfard, A. (2012). Spontaneous meta-arithmetic as a first step towards school algebra. International Journal of Educational Research. 51-52, 45-65

Daro, P., F. Mosher, T. Corcoran. (2011). *Learning trajectories in mathematics: A foundation for standards, curriculum, assessment, and instruction*. Consortium for Policy Research in Education.

Ronda, E. (2004). A Framework of Growth Points in students' developing understanding of function. Unpublished PHD: Australian Catholic University.

Ronda, E. (2009). Growth Points in Students' Developing understanding of functions in equation form. Mathematics Education Research Journal, 21(1), 31-35.

Wilson, P. H., Mojica, G. F., & Confrey, J. (2013). Learning trajectories in teacher education: Supporting teachers' understandings of students' mathematical thinking. *The Journal of Mathematical Behavior*, *32*(2), 103-121.